Σ-protocols

- and why they should matter to every Bitcoin thinker

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Advancing Bitcoin

Outline

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- **Motivation** Σ-protocols in the wild

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- **ZK** Analogies; zero knowledge
- **Homomorphism** the special property needed for Σ-protocols
- **The most fundamental Σ-protocol**
- **The Fiat-Shamir transform**
- **Combinations of Σ-protocols - AND, OR, ...**

Sigma protocols in the wild

Σ-protocols in the wild - 0

Let `M = rT` be the blinded token that C sends to S, let `(G,Y) = (G,xG)` be the commitment from above, and let H_3 be a new hash function (modelled as a random oracle for security purposes). In the protocol below, we can think of S playing the role of the 'prover' and C the 'verifier' in a traditional NIZK proof system.

- S computes `Z = xM`, as before.
- S also samples a random nonce `k ← ZZ_q` and commits to the nonce by calculating `A = kG` and `B = kM`
- S constructs a challenge `c ← H_3(G,Y,M,Z,A,B)` and computes `s = k-cx (mod q)`
- S sends `(c,s)` to the user C
- C recalculates `A' = sG + cY` and `B' = s*M + c*Z` and hashes `c' = H_3(G,Y,M,Z,A',B')`.
- C verifies that `c' =?= c`.

Note that correctness follows since

```
A' = sG + cY = (k-cx)G + cxG = kG and B' = sM + cZ = r(k-cx)T + crxT = krT = kM
```

We write DLEQ(Z/M == Y/G) to denote the proof that is created by S and validated by C.

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Anonymous credentials (used in e.g. Brave, Wabisabi, Signal)

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Core concepts:
Proving without revealing (ZKPs)

Zero knowledge, intuitively - 1

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- Coke's secret recipe. You claim to know coke's secret recipe? ok, here's 10000 ingredients in a kitchen. I'll walk away for a day but keep you locked in the kitchen. Make me a glass of Coke.

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- To win at this game, you have to *be ready to create a demonstration for any challenge*.
- But your demonstration shouldn't give away the secret sauce .. 🤔

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- Since there's a big incentive to cheat, this wouldn't work over a telephone call, because the side who reveals their (choice or flip)*second* can always win.
- This example illustrates the idea of a **commitment** - hand fixes and hides the coin, that's a commitment.

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$2 \cdot (a + b) \equiv 2 \cdot a + 2 \cdot b$. Why is this *so* important?

Cryptography: just encrypt/hide?

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$a \cdot G + b \cdot G = c \cdot G \leftarrow a + b = c$

The canonical Σ-protocol

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Schnorr ID protocol

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$\mathcal{P} \Longrightarrow$ "response" $\Longrightarrow \mathcal{V}$.

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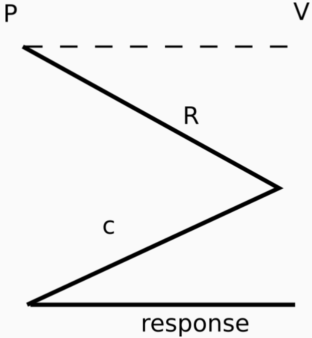

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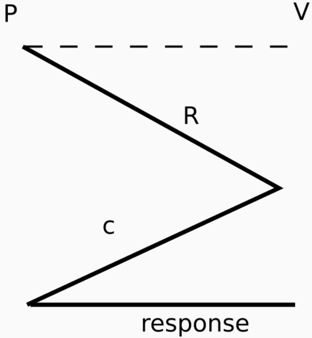

(EC)DL only? RSA, lattices - anything with homomorphism.

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Exactly because there is a homomorphism

for EC point addition, it works!

The Fiat-Shamir transform

 

 

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Domain separation tags? See BIP340 $\mathbb{H}_{tag}()$

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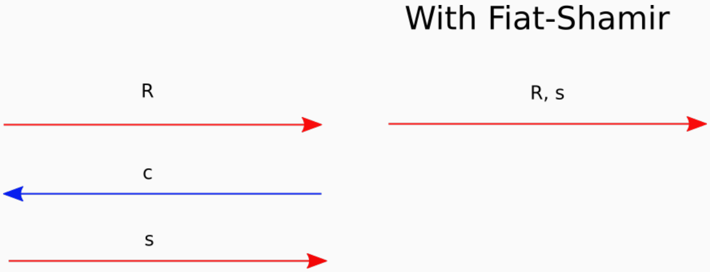

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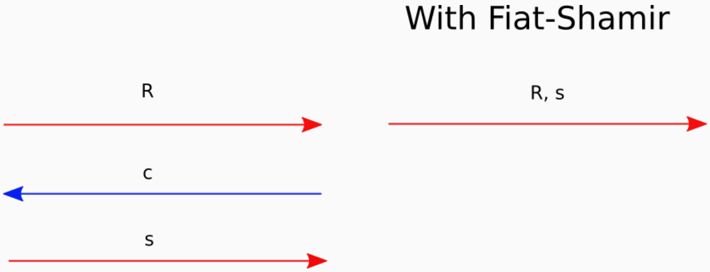

Note that $\mathcal{V}$ must be able to recreate c as the hash (of R, etc.)

Increasing the power level: COMBINING Σ-protocols

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Answer: $R_1, R_2, P_1, P_2, \ldots$.

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$\mathcal{V}$: $s_n \cdot G \stackrel{?}{=} R_n + c_n \cdot P_n \ \wedge \ c \stackrel{?}{=} c_1 \oplus c_2$

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AOS style is different: form a causal loop over the whole set of 4 by each challenge hashing the *previous* index. More efficient.

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$\mathcal{V}$ checks: $s \cdot G \stackrel{?}{=} R_1 + c \cdot P \;\wedge\; s \cdot H \stackrel{?}{=} R_2 + c \cdot Q$.

Many keys in linear relationships

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$3s_1 + 10s_2 \stackrel{?}{=} 15c \wedge c \stackrel{?}{=} \mathbb{H}(P_1, P_2, R_1, R_2, G_1, G_2)$.

Can generalize to a whole set of linear simultaneous equations

Conclusion

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- (See: "Okamoto's protocol for representations".)

References - 1

- Boneh and Shoup, see Chapter 19
- Dan Boneh on Σ-protocols - video lecture
- standardisation of Σ-protocols (survey)
- concept of DLEQ
- explainer on DLEQs
- Nadav Kohen on blind sigs

References - 2

- How PrivacyPass uses DLEQs
- My Bulletproofs writeup
- Camenisch & Stadler '97 Proof systems for discrete logs
- Ring signatures - my blog
- Matt Green on PAKE
- Brands' book on credentials (see also: uprove)

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- Chaum's original blind signatures paper
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Thank you

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